Hamiltonian Cosmological Perturbation Theory

B.M. Barbashov, [a] V.N. Pervushin, [a] A.F. Zakharov, [b,a,c,d] V.A. Zinchuk [a]

[a] *Bogoliubov Laboratory for Theoretical Physics, JINR, 141980 Dubna, Russia*

[b] *National Astronomical Observatories of Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing, 100012, China*

[c] *Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya, 25, 117259, Moscow, Russia*

[d] *Astro Space Center of Lebedev Physics Institute of RAS, Moscow*

Abstract

The Hamiltonian approach to cosmological perturbations in general relativity in finite space-time is developed, where a cosmological scale factor is identified with spatial averaging the metric determinant logarithm. This identification preserves the number of variables and leads to a cosmological perturbation theory with the scalar potential perturbations in contrast to the kinetic perturbations in the Lifshitz version which are responsible for the "primordial power spectrum" of CMB in the inflationary model. The Hamiltonian approach enables to explain this "spectrum" in terms of scale-invariant variables and to consider other topical problem of modern cosmology in the context of quantum cosmological creation of both universes and particles from the stable Bogoliubov vacuum.

Key words: General Relativity and Gravitation, Cosmology, Observational Cosmology

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1. Introduction

The cosmological perturbation theory in general relativity (GR) [1,2] based on the separation of the cosmological scale factor by the transformation $g_{\mu\nu} = a^2\widetilde{g}_{\mu\nu}$ is one of the basic tools applied for analysis of modern observational data including including Cosmic Microwave Background (CMB).

In the present paper we discuss the problem of the relation between the cosmological perturbation theory and the Hamiltonian approach [3,4] to GR, where a similar scale factor was considered in [5] as the homogeneous invariant evolution parameter in accordance with the Hamiltonian diffeomorphism subgroup $x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0)$ [6] meaning in fact that the coordinate evolution parameter x^0 is not observable. The statement of the problem is to formulate the cosmological perturbation theory in terms of diffeo-invariant quantities.

The content of the paper is the following. In Section 2, the statement of the problem is given. In Section 3, it is shown that the separation of the scale factor can lead to exact resolution of the energy constraint in GR and to its Hamiltonian reduction. Sections 4 and 5 are devoted to cosmological models of both classical and quantum universes that follows from the reduced theory. The Hamiltonian perturbation theory and its comparison with Lifshitz's one are given in Section 6.

2. Statement of problem

GR is given in terms of metric components and fields f by the Hilbert action

$$S = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \mathcal{L}_{(\mathrm{M})}(\varphi_0|g,f)\right] \tag{1}$$

and the space-time geometric interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where the parameter $\varphi_0 = \sqrt{3/8\pi G_0} = \sqrt{3M^2_{\mathrm{Planck}}/8\pi}$ scales all masses, and G_0 is the Newton coupling constant in units $\hbar = c = 1$. The Hamiltonian approach is formulated by means of a geometric interval

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \equiv \omega_{(0)}\omega_{(0)} - \omega_{(1)}\omega_{(1)} - \omega_{(2)}\omega_{(2)} - \omega_{(3)}\omega_{(3)}, \tag{2}$$

where $\omega_{(\alpha)}$ are linear differential forms [7] in terms of the Dirac variables [3]

$$\omega_{(0)} = \psi^6 N_d dx^0, \qquad \omega_{(b)} = \psi^2\mathbf{e}_{(b)i}(dx^i + N^i dx^0); \tag{3}$$

here ψ is the spatial metrics determinant, $\mathbf{e}_{(a)i}$ are triads with $\det|\mathbf{e}| = 1$, N_d is the Dirac lapse function, and N^i is the shift vector. The comparison of this interval with the one

$$ds^2 = a^2(\eta)\left[(1+2\Phi)d\eta^2 - 2N_k dx^k d\eta - (1-2\Psi)(dx^k)^2 - dx^i dx^j(h_{ij})\right] \tag{4}$$

used in the cosmological perturbation theory [1] raises to the following question: Is it possible to formulate the Hamiltonian approach to GR in terms of the metric components (3) so that the conformal time η in Eq. (4) as the measurable one of a cosmic photons becomes diffeo-invariant quantity with respect to the time-coordinate transformations $x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0)$, and the potential Ψ does not contain one more a homogeneous component, in order to preserve the number of variables of GR?

3. Separation of Scale Factor and Hamiltonian Reduction

The invariance of the action (1) and interval (2) expressed in terms of the Fock – Dirac simplex (3) with respect to time-coordinate transformations $x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0)$ means that a diffeo-invariant "evolution parameter" in GR coincides with one of homogeneous variables [5,6,8]. The cosmological evolution is the irrefutable observational argument in favor of existence of such a homogeneous variable considered in GR as the cosmological scale factor. The cosmological scale factor $a(x_0)$ introduced by the scale transformation: $g_{\mu\nu} = a^2(x_0)\widetilde{g}_{\mu\nu}$, where $\widetilde{g}_{\mu\nu}$ is defined by (2) and (3), where $\widetilde{N}_d = a^2 N_d$ and $\widetilde{\psi}^2 = a^{-1}\psi^2$. In order to keep the number of variables of GR, the scale factor can be defined using the spatial averaging $\log\sqrt{a} \equiv \langle \log\psi\rangle \equiv V_0^{-1}\int d^3x \log\psi$, so that the rest scalar component $\widetilde{\psi}$ satisfies the identity

$$\langle \log\widetilde{\psi}\rangle \equiv V_0^{-1}\int d^3x \log\widetilde{\psi} = V_0^{-1}\int d^3x \left[\log\psi - \langle\log\psi\rangle\right] \equiv 0, \tag{5}$$

where $V_0 = \int d^3x$ is finite volume. The similar scale transformation of a curvature $\sqrt{-g}R(g) = a^2\sqrt{-\widetilde{g}}R(\widetilde{g}) - 6a\partial_0\left[\partial_0 a\sqrt{-\widetilde{g}}\;\widetilde{g}^{00}\right]$ converts action (1) into

$$S = \widetilde{S} - \int\limits_{V_0} dx^0(\partial_0\varphi)^2\int d^3x\widetilde{N}_d^{-1}, \tag{6}$$

where $\widetilde{S}$ is the action (1) in terms of metrics $\widetilde{g}$ and the running scale of all masses $\varphi(x^0) = \varphi_0 a(x^0)$ and $(\widetilde{N}_d)^{-1} = \sqrt{-\widetilde{g}}\;\widetilde{g}^{00}$. One can construct the Hamiltonian function using the definition of a set of canonical momenta:

$$P_\varphi = \frac{\partial L}{\partial(\partial_0\varphi)} = -2V_0\partial_0\varphi\left\langle(\widetilde{N}_d)^{-1}\right\rangle = -2V_0\frac{d\varphi}{d\zeta} \equiv -2V_0\varphi', \tag{7}$$

$$p_\psi = \frac{\partial\mathcal{L}}{\partial(\partial_0\log\widetilde{\psi})} \equiv -\frac{4\varphi^2}{3}\cdot\frac{\partial_l(\widetilde{\psi}^6 N^l) - \partial_0(\widetilde{\psi}^6)}{\widetilde{\psi}^6\widetilde{N}_d}, \tag{8}$$

where $d\zeta = \langle(\widetilde{N}_d)^{-1}\rangle^{-1}dx^0$ is a time-interval invariant with respect to time-coordinate transformations $x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0)$. One can construct the Hamiltonian form of the action in terms of momenta P_φ and $P_F = [p_\psi, p^i_{(b)}, p_f]$ including (7), (8)

$$S = \int dx^0 \left[\int d^3x \left(\sum_F P_F\partial_0 F + C - \widetilde{N}_d\widetilde{T}^0_0\right) - P_\varphi\partial_0\varphi + \frac{P^2_\varphi}{4\int dx^3(\widetilde{N}_d)^{-1}}\right], \quad (9)$$

where $\mathcal{C} = N^i T^0_i + C_0 p_\psi + C_{(b)}\partial_k \mathbf{e}^k_{(b)}$ is the sum of constraints with the Lagrangian multipliers $N^i, C_0,\ C_{(b)}$ and the energy–momentum tensor components T^0_i; these constraints include the transversality $\partial_i \mathbf{e}^i_{(b)} \simeq 0$ and the Dirac minimal surface [3]:

$$p_\psi \simeq 0 \quad \Rightarrow \quad \partial_j(\widetilde{\psi}^6\mathcal{N}^j) = (\widetilde{\psi}^6)' \qquad (\mathcal{N}^j = N^j\langle\widetilde{N}_d^{-1}\rangle). \quad (10)$$

The explicit dependence of $\widetilde{T}^0_0$ on $\overline{\psi}$ can be given in terms of the scale-invariant Lichnerowicz variables [8] $\omega^{(L)}_{(\mu)} = \psi^{-2}\omega_{(\mu)}$:

$$\widetilde{T}^0_0 = \widetilde{\psi}^7\hat{\triangle}\widetilde{\psi} + \sum_I \widetilde{\psi}^I a^{I/2-2}\tau_I, \quad (11)$$

where $\hat{\triangle}F \equiv (4\varphi^2/3)\partial_{(b)}\partial_{(b)}F$ is the Laplace operator and τ_I is partial energy density marked by the index I running a set of values $I = 0$ (stiff), 4 (radiation), 6 (mass), 8 (curvature), 12 (Λ-term) in accordance with a type of matter field contributions, and a is the scale factor.

The energy constraint $\delta S[\varphi_0]/\delta\widetilde{N}_d = 0$ takes the algebraic form

$$-\frac{\delta\widetilde{S}[\varphi]}{\delta\widetilde{N}_d} \equiv \widetilde{T}^0_0 = \frac{(\partial_0\varphi)^2}{\widetilde{N}^2_d} = \frac{P^2_\varphi}{4V_0^2[\langle(\widetilde{N}_d)^{-1}\rangle\widetilde{N}_d]^2}, \quad (12)$$

where T^0_0 is the local energy density by definition. The spatial averaging of this equation multiplied by $\widetilde{N}_d$ looks like the energy constraint

$$P^2_\varphi = E^2_\varphi, \quad (13)$$

where the Hamiltonian functional $E_\varphi = 2\int d^3x(\widetilde{T}^0_0)^{1/2} = 2V_0\langle(\widetilde{T}^0_0)^{1/2}\rangle$ can be treated as the "universe energy" by analogy with the "particle energy" in special relativity (SR). Eqs. (7) and (13) have the exact solution

$$\zeta(\varphi_0|\varphi) \equiv \int dx^0 \left\langle(\widetilde{N}_d)^{-1}\right\rangle^{-1} = \pm\int\limits_{\varphi}^{\varphi_0} d\widetilde{\varphi}\left\langle(\widetilde{T}^0_0(\widetilde{\varphi}))^{1/2}\right\rangle^{-1} \quad (14)$$

well known as the Hubble-type evolution. The local part of Eq. (12) determines a part of the Dirac lapse function invariant with respect to diffeomorphisms of the Hamiltonian formulation $x^0 \to \widetilde{x}^0 = \widetilde{x}^0(x^0)$ [6]:

$$N_{\rm inv} = \langle (\widetilde{N}_d)^{-1} \rangle \widetilde{N}_d = \left\langle \sqrt{\widetilde{T}_0^0} \right\rangle \left(\sqrt{\widetilde{T}_0^0} \right)^{-1}. \tag{15}$$

One can find evolution of all field variables $F(\varphi, x^i)$ with respect to φ by the variation of the "reduced" action obtained as values of the Hamiltonian form of the initial action (9) onto the energy constraint (13) [5]:

$$S|_{P_\varphi = \pm E_\varphi} = \int\limits_{\varphi}^{\varphi_0} d\widetilde{\varphi} \left\{ \int d^3x \left[\sum_F P_F \partial_\varphi F + \bar{\mathcal{C}} \mp 2\sqrt{\widetilde{T}_0^0(\widetilde{\varphi})} \right] \right\}, \tag{16}$$

where $\bar{\mathcal{C}} = \mathcal{C}/\partial_0\widetilde{\varphi}$. The reduced Hamiltonian $\sqrt{\widetilde{T}_0^0}$ is Hermitian, if the minimal surface constraint (10) removes a negative contribution of p_ψ from the energy density [9]. The reduced action (16) shows us that the initial data at the beginning $\varphi = \varphi_I$ are independent of the present-day ones at $\varphi = \varphi_0$, therefore the proposal about an existence of the Planck epoch $\varphi = \varphi_0$ at the beginning [2] looks very doubtful. Let us consider consequences of the classical reduced theory (16) and quantization of the energy constraint (13) without the "Planck epoch" at the beginning.

4. Observational Data in Terms of Scale-Invariant Variables

Let us assume that the local density $T_0^0 = \rho_{(0)}(\varphi) + T_{\rm f}$ contains a tremendous cosmological background $\rho_{(0)}(\varphi)$. The low-energy decomposition of "reduced" action (16) $2d\varphi\sqrt{\widetilde{T}_0^0} = 2d\varphi\sqrt{\rho_{(0)} + T_{\rm f}} = d\varphi\left[2\sqrt{\rho_{(0)}} + T_{\rm f}/\sqrt{\rho_{(0)}}\right] + ...$ over field density $T_{\rm f}$ gives the sum $S|_{P_\varphi = +E_\varphi} = S^{(+)}_{\rm cosmic} + S^{(+)}_{\rm field} + \ldots$, where the first term of this sum $S^{(+)}_{\rm cosmic} = +2V_0 \int\limits_{\varphi_I}^{\varphi_0} d\varphi\sqrt{\rho_{(0)}(\varphi)}$ is the reduced cosmological action, whereas the second one is the standard field action of GR and SM

$$S^{(+)}_{\rm field} = \int\limits_{\zeta_I}^{\zeta_0} d\zeta \int d^3x \left[\sum_F P_F \partial_\eta F + \bar{\mathcal{C}} - T_{\rm f} \right] \tag{17}$$

in the space determined by the interval

$$ds^2 = d\zeta^2 - [e_{(a)i}(dx^i + \mathcal{N}^i d\zeta]^2; \quad \partial_i e^i_{(a)} = 0, \quad \partial_i \mathcal{N}^i = 0 \tag{18}$$

with conformal time $d\eta = d\zeta = d\varphi/\rho_{(0)}^{1/2}$ as the diffeo-invariant and scale-invariant quantity, coordinate distance $r = |x|$, and running masses $m(\zeta) = a(\zeta)m_0$. We see that the correspondence principle leads to the theory (17), where the scale-invariant conformal variables and coordinates are identified with observable ones and the cosmic evolution with the evolution of masses:

$$\frac{E_{\rm emission}}{E_0} = \frac{m_{\rm atom}(\eta_0 - r)}{m_{\rm atom}(\eta_0)} = \frac{\varphi(\eta_0 - r)}{\varphi_0} = a(\eta_0 - r) = \frac{1}{1+z}.$$

The conformal observable distance r loses the factor a, in comparison with the nonconformal one $R = ar$. Therefore, in this case, the redshift – coordinate-distance relation $d\eta = d\varphi/\sqrt{\rho_0(\varphi)}$ corresponds to a different equation of state than in the standard one [10,11]. The best fit to the data including cosmological SN observations [12] requires a cosmological constant $\Omega_\Lambda = 0.7$, $\Omega_{\rm CDM} = 0.3$ in the case of the Friedmann "scale-variant quantities" of standard cosmology, whereas for the "scale-invariant conformal quantities" these data are consistent with the dominance of the stiff state of free scalar field $\Omega_{\rm Stiff} = 0.85 \pm 0.15$, $\Omega_{\rm CDM} = 0.15 \pm 0.10$ [10]. If $\Omega_{\rm Stiff} = 1$, we have the square root dependence of the scale factor on conformal time $a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}$. Just this time dependence of the scale factor on the measurable time (here – conformal one) is used for description of the primordial nucleosynthesis [11,13]. Thus the stiff state can describe all epochs including the creation of a quantum universe.

5. The Quantum Universe

We have seen above that in the "reduced" action (16) momenta $P_{\varphi\pm} = \pm E_\varphi$ become the generators of evolution of all variables with respect to the evolution parameter φ [5] forward and backward, respectively. The negative energy problem can be solved by the primary quantization of the energy constraint $[P_\varphi^2 - E_\varphi^2]\Psi_{\rm u} = 0$ and the secondary quantization $\Psi_{\rm u} = (1/\sqrt{2E_\varphi})[A^+ + A^-]$ by the Bogoliubov transformation $A^+ = \alpha B^+ + \beta^* B^-$, in order to diagonalize the equations of motion by the condensation of "universes" $< 0|\frac{i}{2}[A^+A^+ - A^-A^-]|0 >= R(\varphi)$ and describe cosmological creation of a "number" of universes $< 0|A^+A^-|0 >= N(\varphi)$ from the stable Bogoliubov vacuum $B^-|0 >= 0$. The vacuum postulate $B^-|0 >= 0$ leads to an arrow of the conformal time $\eta \geq 0$ and its absolute point of reference $\eta = 0$ at the moment of creation $\varphi = \varphi_I$ [5]. The cosmological creation of the *"universes"* is described by the Bogoliubov equations [14]

$$\frac{dN}{d\varphi} = \frac{dE_\varphi}{2E_\varphi d\varphi}\sqrt{4N(N+1) - R^2}, \quad \frac{dR}{d\varphi} = -2E_\varphi\sqrt{4N(N+1) - R^2}.$$

In the model of the stiff state $\rho = p$, where $E_\varphi = Q/\varphi$, these equations have an solution [14] $N(\varphi) = \frac{1}{4Q}R(\varphi) = \frac{1}{4Q^2-1}\sin^2\left[\sqrt{Q^2-\frac{1}{4}}\ \ \ln\frac{\varphi}{\varphi_I}\right] \neq 0$, where $\varphi = \varphi_I\sqrt{1+2H_I\eta}$. The initial data $\varphi_I = \varphi(\eta = 0), H_I = \varphi_I'/\varphi_I = Q/(2V_0\varphi_I^2)$ are considered as a point of creation or annihilation of a universe; whereas the Planck value of the running mass scale $\varphi_0 = \varphi(\eta = \eta_0)$ belongs to the present day moment η_0.

These initial data φ_I and H_I are determined by parameters of matter cosmologically created from the Bogoliubov vacuum at the beginning of a universe $\eta \simeq 0$. In the Standard Model (SM), W-, Z- vector bosons have maximal probability of this cosmological creation due to their mass singularity [15]. The uncertainty principle $\triangle E \cdot \triangle\eta \geq 1$ (where $\triangle E = 2M_I, \triangle\eta = 1/2H_I$) shows that at the moment of creation of vector bosons their Compton length defined by its inverse mass $M_{\rm I}^{-1} = (a_{\rm I}M_{\rm W})^{-1}$ is close to the universe horizon defined in the stiff state as $H_{\rm I}^{-1} = a_{\rm I}^2(H_0)^{-1}$. Equating these quantities $M_{\rm I} = H_{\rm I}$ one can estimate the initial data of the scale factor $a_{\rm I}^2 = (H_0/M_{\rm W})^{2/3} = 10^{-29}$ and the Hubble parameter $H_{\rm I} = 10^{29}H_0 \sim 1\ {\rm mm}^{-1} \sim 3K$. Just at this moment there is an effect of intensive cosmological creation of the vector bosons described in [15]; in particular, the distribution functions of the longitudinal vector bosons demonstrate a large contribution of relativistic momenta. Their temperature T_c can be estimated from the equation in the kinetic theory for the time of establishment of this temperature $\eta^{-1}_{relaxation} \sim n(T_c) \times \sigma \sim H$, where $n(T_c) \sim T_c^3$ and $\sigma \sim 1/M_I^2$ is the cross-section. This kinetic equation and values of the initial data $M_{\rm I} = H_{\rm I}$ give the temperature of relativistic bosons $T_c \sim (M_{\rm I}^2H_{\rm I})^{1/3} = (M_0^2H_0)^{1/3} \sim 3K$ as a conserved number of cosmic evolution compatible with the SN data [10]. We can see that this value is surprisingly close to the observed temperature of the CMB radiation $T_c = T_{\rm CMB} = 2.73$ K. The primordial mesons before their decays polarize the Dirac fermion vacuum and give the baryon asymmetry frozen by the CP – violation so that $n_b/n_\gamma \sim X_{CP} \sim 10^{-9}$, $\Omega_b \sim \alpha_{\rm QED}/\sin^2\theta_{\rm Weinberg} \sim 0.03$, and $\Omega_R \sim 10^{-5} \div 10^{-4}$ [15]. All these results testify to that all visible matter can be a product of decays of primordial bosons, and the observational data on CMB can reflect parameters of the primordial bosons, but not the matter at the time of recombination. The length of the semi-circle on the surface of the last emission of photons at the life-time of W-bosons in terms of the length of an emitter (i.e. $M_W^{-1}(\eta_L) = (\alpha_W/2)^{1/3}(T_c)^{-1}$) is $\pi \cdot 2/\alpha_W$. It is close to $l_{min} \sim 210$ of CMB, whereas $(\triangle T/T)$ is proportional to the inverse number of emitters $(\alpha_W)^3 \sim 10^{-5}$. The temperature history of the expanding universe in this case looks like the history of evolution of masses of elementary particles in the cold universe with the constant conformal temperature $T_c = a(\eta)T = 2.73$ K of the cosmic microwave background. The equations describing the longitudinal vector bosons in SM, in this case, are close to the equations that are used, in the inflationary model [2], for description of the "power primordial spectrum" of the CMB radiation.

6. The Potential Perturbations and Shift Vector

In order to simplify equations of the scalar potentials $N_{\rm inv}, \widetilde{\psi}$, one can introduce a new table of symbols: $N_{\rm s} = \psi^7 N_{\rm inv}$, $\widetilde{T} = \sum_I \widetilde{\psi}^{(I-7)} a^{\frac{I}{2}-2}\tau_I$, $\rho_{(0)} = \langle (\widetilde{T}_0^0)^{1/2}\rangle^2 = \varphi'^2$. The variations of the action (9) with respect to $N_{\rm s}, \log\widetilde{\psi}$ lead to equations

$$\hat{\triangle}\widetilde{\psi} + \widetilde{T} = \frac{\widetilde{\psi}^{\ 7}\rho_{(0)}}{N_{\rm s}^2}, \tag{19}$$

$$\widetilde{\psi}\hat{\triangle}N_{\rm s} + N_{\rm s}\frac{d\widetilde{T}}{d\log\widetilde{\psi}} + 7\frac{\widetilde{\psi}^{\ 7}\rho_{(0)}}{N_{\rm s}} = \rho_{(1)}, \tag{20}$$

respectively, where $\rho_{(1)} = \langle \widetilde{\psi}\hat{\triangle}N_{\rm s} + N_{\rm s}\widetilde{\psi}\partial_{\widetilde{\psi}}\widetilde{T} + 7\widetilde{\psi}^{\ 7}\rho_{(0)}/N_{\rm s}\rangle$.

For $N_{\rm s} = 1 - \nu_1$ and $\widetilde{\psi} = 1 + \mu_1$ and the small deviations $\mu_1, \nu_1 \ll 1$ the first orders of Eqs. (19) and (20) take the form

$$[-\hat{\triangle} + 14\rho_{(0)} - \rho_{(1)}]\mu_1 + 2\rho_{(0)}\nu_1 = \overline{\tau}_{(0)}, \tag{21}$$

$$[7\cdot 14\rho_{(0)} - 14\rho_{(1)} + \rho_{(2)}]\mu_1 + [-\hat{\triangle} + 14\rho_{(0)} - \rho_{(1)}]\nu_1 = 7\overline{\tau}_{(0)} - \overline{\tau}_{(1)}, \tag{22}$$

where $\rho_{(n)} = \langle \tau_{(n)}\rangle \equiv \sum_I I^n a^{\frac{I}{2}-2}\langle \tau_I\rangle$. This choice of variables determines $\widetilde{\psi} = 1 + \mu_1$ and $N_{\rm s} = 1 - \nu_1$ in the form of a sum

$$\widetilde{\psi} = 1 + \frac{1}{2}\int d^3y \left[D_{(+)}(x,y)\overline{T}^{(\mu)}_{(+)}(y) + D_{(-)}(x,y)\overline{T}^{(\mu)}_{(-)}(y)\right], \tag{23}$$

$$N_{\rm s} = 1 - \frac{1}{2}\int d^3y \left[D_{(+)}(x,y)\overline{T}^{(\nu)}_{(+)}(y) + D_{(-)}(x,y)\overline{T}^{(\nu)}_{(-)}(y)\right], \tag{24}$$

where β is given by

$$\beta = \sqrt{1 + [\langle \tau_{(2)}\rangle - 14\langle \tau_{(1)}\rangle]/(98\langle \tau_{(0)}\rangle)}, \tag{25}$$

$$\overline{T}^{(\mu)}_{(\pm)} = \overline{\tau_{(0)}} \mp 7\beta[7\overline{\tau_{(0)}} - \overline{\tau_{(1)}}], \qquad \overline{T}^{(\nu)}_{(\pm)} = [7\overline{\tau_{(0)}} - \overline{\tau_{(1)}}] \pm (14\beta)^{-1}\overline{\tau_{(0)}} \tag{26}$$

are the local currents, $D_{(\pm)}(x,y)$ are the Green functions satisfying the equations

$$[\pm\hat{m}^2_{(\pm)} - \hat{\triangle}]D_{(\pm)}(x,y) = \delta^3(x-y), \tag{27}$$

where $\hat{m}^2_{(\pm)} = 14(\beta \pm 1)\langle \tau_{(0)}\rangle \mp \langle \tau_{(1)}\rangle$.

In the case of point mass distribution in a finite volume V_0 with the zero pressure and the density $\overline{\tau_{(0)}}(x) = \overline{\tau_{(1)}}(x)/6 \equiv M\,[\delta^3(x-y) - 1/V_0]$, solutions (23), (24) take a form

$$\widetilde{\psi} = 1 + \mu_1 = 1 + \frac{r_g}{4r}\left[\gamma_1 e^{-m_{(+)}(z)r} + (1-\gamma_1)\cos m_{(-)}(z)r\right], \qquad (28)$$
$$N_{\rm s} = 1 - \nu_1 = 1 - \frac{r_g}{4r}\left[(1-\gamma_2)e^{-m_{(+)}(z)r} + \gamma_2 \cos m_{(-)}(z)r\right], \qquad (29)$$

where $\gamma_1 = \frac{1+7\beta}{2}$, $\gamma_2 = \frac{14\beta-1}{28\beta}$, $r_g = \frac{3M}{4\pi\varphi^2}$, $r = |x-y|$, $m^2_{(\pm)} = \frac{3\hat{m}^2_{(\pm)}}{4\varphi^2}$.

The minimal surface (9) $\partial_i[\overline{\psi}^6 \mathcal{N}^i] - (\overline{\psi}^6)' = 0$ gives the shift of the coordinate origin in the process of evolution

$$\mathcal{N}^i = \left(\frac{x^i}{r}\right)\left(\frac{\partial_\zeta V}{\partial_r V}\right), \qquad V(\zeta, r) = \int^r d\tilde{r}\; \tilde{r}^2 \widetilde{\psi}^6(\zeta, \tilde{r}). \qquad (30)$$

Solutions (28), (29) have spatial oscillations and the nonzero shift of the coordinate origin of the type of (30). In the infinite volume limit $\langle \tau_{(n)} \rangle = 0,\ a = 1$ solutions (28) and (29) coincide with the isotropic version of the Schwarzschild solutions: $\widetilde{\psi} = 1 + \frac{r_g}{4r}$, $N_{\rm s} = 1 - \frac{r_g}{4r}$, $N^k = 0$.

Now one can compare the Hamiltonian perturbation theory with the standard cosmological perturbation theory (4) [1] where $\overline{\Phi} = \nu_1 + \mu_1$, $\overline{\Psi} = 2\mu_1$, $N^i = 0$. We note that the zero-Fourier harmonics of the spatial determinant is taken into account in [2] twice that is an obstruction to the Hamiltonian method. The Hamiltonian approach shows us that if this double counting is removed, the equations of scalar potentials (21), and (22)) will not contain time derivatives that are responsible for the CMB "primordial power spectrum" in the inflationary model [2]. However, these equations of the Lifshits perturbation theory are close to ones of the primordial vector bosons. We have seen above that the Hamiltonian approach to GR gives us another possibility to explain the CMB "spectrum" by cosmological creation of vector W-, Z- bosons.

The next differences are a nonzero shift vector and the spatial oscillations of the scalar potentials determined by $m^2_{(-)}$. In the diffeo-invariant version of cosmology [10], the SN data dominance of stiff state determines the parameter of spatial oscillations $m^2_{(-)} = \frac{6}{7}H_0^2[\Omega_{\rm R}(z+1)^2 + \frac{9}{2}\Omega_{\rm Mass}(z+1)]$. The redshifts in the recombination epoch $z_r \sim 1100$ and the clustering parameter $r_{\rm clustering} = \pi/m_{(-)} \sim \pi/[H_0\Omega_R^{1/2}(1+z_r)] \sim 130\,{\rm Mpc}$ recently discovered in the researches of large scale periodicity in redshift distribution [16] lead to a reasonable value of the radiation-type density $10^{-4} < \Omega_R \sim 3 \cdot 10^{-3}$ at the time of this epoch.

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